Network Design with Elastic Demand and Dynamic Passenger Assignment to Assess the Performance of Transit Services

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Abstract: This study proposes a solution framework for operational analysis and financial assessment of transit services that considers the passenger behavior and the elasticity of transit demand to service characteristics. The proposed solution framework integrates a dynamic transit passenger assignment model (Fast-Trips) with a mode choice model and a service design module, and iterates these methods until an equilibrium between fares and frequencies is reached. The solution framework was implemented for a newly conceived intercity transit service in Arizona, and the system performance was studied for multiple fare policy and frequency design scenarios. The results showed that the scenarios with designed-oriented frequencies had lower ratios of revenue to operating cost (R/C) compared with those in which frequencies were set based on the passenger path-choice behaviors and route usage, which emphasizes the importance of considering elastic transit demand in network and service designs. The sensitivity analysis also indicated that there are multiple ways to achieve a certain R/C ratio, and therefore it is the other objectives and the operator's priorities that define the final design and service characteristics.
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Introduction

Transit operators face questions such as how much the fare should be to make the system profitable, how frequently the service should operate so that it does not lose passengers while controlling expenses, or what trade-offs between fares and frequencies should be made to achieve a certain net revenue. Although most public transit (PT) systems are not profitable, it is very important for a system to be financially viable and to maintain an acceptable ratio of revenue-to-operating cost.

Transit operating costs are largely a function of transit routes' travel time and their frequencies, which reflect operating costs per vehicle-hour of travel. On the other hand, revenue is a function of fares and the total number of passengers attracted to the system. However, transit demand is innately elastic, meaning that any change in the service will be followed by a change in the demand, and vice versa. In other words, service characteristics such as fares, travel time, and frequencies influence the number of passengers attracted to the system, whereas the number of attracted passengers itself will influence the routes' design and frequencies. This makes it even more challenging to answer the preceding questions and to assess the financial viability of a transit service. Therefore, to have a more accurate financial assessment, it is critical to consider the elasticity of transit demand to service frequencies and fare policies in designing a transit service.

The elasticity of transit demand to routes and their frequencies and its effect on transit network design have been studied by many researchers (e.g., Lee and Vuchic 2005; Fan and Machemehl 2006; Ranjbari et al. 2012). However, those studies focused only on the network design and did not consider fares, which is a critical factor in travel choice behavior and directly influences the number of passengers attracted to the system. Moreover, the passenger assignment to routes in those studies was all-or-nothing or another simple static assignment; however, in order to truly capture the passenger behaviors, it is important to have a dynamic and passenger-level assignment model that captures the route choice behavior of passengers and the fare that they pay.

Dynamic transit assignment models are relatively new in the literature, and started with the introduction of schedule-based models in the last two decades (Nuzzolo and Russo 1996; Nielsen and Jovicic 1999; Hamdouch and Lawphongpanich 2008; Hickman and Bernstein 1997; Nuzzolo et al. 2001). As opposed to frequency-based transit assignment models, which consider the average performance of a transit route, schedule-based models can capture the dynamics of the system within periods by modeling every transit vehicle trip separately, and more information about the transit system makes it possible to consider analytic route choice behavior for the transit users. Another significant improvement in transit modeling was the introduction of strategy-based (Spiess and Florian 1989) or hyperpath-based (Nguyen and Pallottino 1988; De Cea and Fernandez 1989) models as a new class of transit route choice models, which consider a set of attractive routes for the transit users, who can choose a route from among this set that minimizes their overall travel time. The adaptation of the

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hyperpath approach to schedule-based modeling was studied by Nguyen et al. (1998, 2001), Noh et al. (2012), and Khani et al. (2015). In these cases, a set of attractive vehicle trips can be determined for transit users according to the schedule.

The present study proposes a solution framework for operational analysis and financial assessment of transit services. In contrast to previous studies, this study considers the elastic transit demand by integrating a dynamic transit passenger assignment model with a mode choice model and a service design module in an iterative approach (Fig. 1). The mode choice model should be sensitive to service characteristics, such as travel time, travel cost, and service frequencies, so that every time these characteristics change, transit demand is updated. The transit assignment model considered for this study is a schedule-based and capacity-constrained model that considers hyperpaths in transit route choice called Fast-Trips, standing for **F**lexible **A**ssignment and **S**imulation **T**ool for **Tr**ansit and **I**ntermodal **P**assenger**s** (MTC, SFCTA, and PSRC 2018). Fast-Trips (FT) has high resolution to capture passenger behavior, which makes it possible to consider the fare that each passenger has paid and the route(s) that they have used. The service design module modifies fares and/or route frequencies in every iteration based on the passenger assignment results. The solution framework iterates the aforementioned steps until an equilibrium between fares and frequencies is reached.

The structure of this paper is as follows. The next section describes the case study context. The mode choice model and the process to generate the corresponding utility function parameters are explained in section "Mode Choice Model." Because of its novelty in the network design context, the dynamic transit passenger assignment model (Fast-Trips) is introduced thereafter, followed by the process to prepare the required network and demand files for the studied transit service. Calculation of revenue and operating costs are described afterwards, and section "Solution Framework" explains the proposed solution framework step by step. We studied the transit system performance for multiple fare policy and frequency design scenarios, and the corresponding results are presented in section "Results and Discussion." Finally, section "Conclusion" summarizes the research findings and presents ideas for possible extensions.

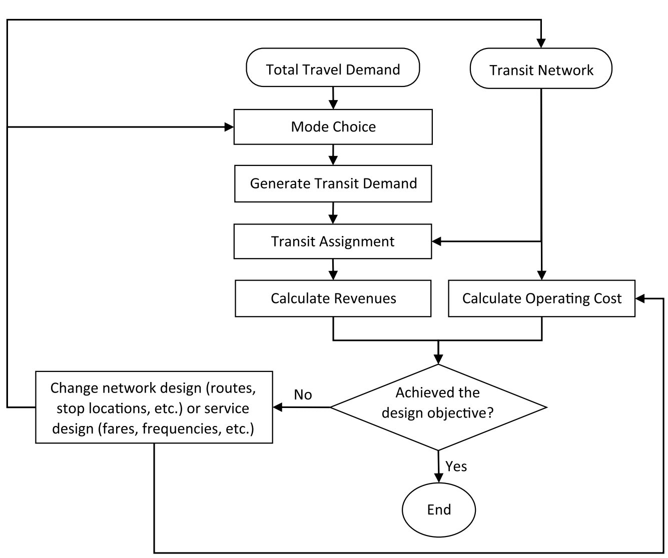

Fig. 1. General solution framework proposed in this study.

Case Study

This study was conducted for a newly conceived intercity transit service between the metropolitan areas of Tucson and Phoenix, Arizona. It is an innovative flexible and high-speed transit service called Flexpress (Ranjbari et al. 2016).

Phoenix has an area of 1,342 km^2 (518 mi^2), a city population of 1.6 million and a metro population of 4.6 million. Tucson is a city with an area of 600 km^2 (231 mi^2) and a population of 530,000, and its metro population is 1 million (US Census Bureau 2017; American Fact Finder 2017). Flexpress was conceived to offer service in tire-based low-profile electric transit vehicles capable of cruising at up to 250 km/h (155 mph) on a dedicated lane on freeways, with multiple terminals in urban areas to provide passengers with higher levels of accessibility to their origins and destinations. In the urban area, the vehicles will be driven by a driver at normal traffic speeds while picking up/dropping off passengers at terminals, but once entering the dedicated guideway, the driver will initiate the autopilot mode and the vehicle will travel at maximum speed until reaching the metro area, where the driver regains control. The two cities are connected by Interstate 10 (I-10), and Flexpress is envisioned to travel on a dedicated traffic lane built on the median of I-10.

The origin–destination (OD) travel demand matrices between Tucson and Phoenix were provided by the Arizona Department of Transportation (ADOT). The total travel demand between the two cities for the morning peak period is 124,571 person-trips. The total transit demand was assumed to be 5% of the total travel demand for each OD, equaling, 6,228 person-trips.

The Flexpress transit network, including routes, terminals and frequencies, was designed through a transit network design and frequency setting (TNDFSP) model, encompassing a three-step solution framework and a mixed integer linear programming optimization model that minimized the sum of total passenger travel time and vehicle deadheading time, considering several design and service constraints (Ranjbari et al. 2020). The Flexpress network was designed for a baseline scenario which has 22 terminals, 30 routes, and serves 60% of the total transit demand, equaling 3,737 person-trips. From the baseline network of 30 routes, 21 routes have the minimum frequency of 4 bus/h, seven have a frequency of 5–10 bus/h, and two have a frequency of at least 20 bus/h.

Mode Choice Model

Two current viable travel modes between Tucson and Phoenix are driving and regular bus (Greyhound). The Greyhound service has one station in Tucson and two stations in the Phoenix area. It operates every 3 h from morning to late evening, and the fare is $1 for the first rider, and increases up to $45 for additional passengers. As a new intercity transit option, the State of Arizona has been investigating the potential for a passenger rail system between Tucson and Phoenix. The selected route alignment for passenger rail would serve the East Valley, sharing right-of-way with the Union Pacific (freight) Railroad, and would run along I-10 south of Eloy into Tucson (Passenger Rail Corridor Study 2014).

The mode choice model and utility functions used in this study were derived from Ranjbari et al. (2017), who conducted a stated preference (SP) survey and built a mode choice model for intercity travel in the studied corridor, considering the aforementioned modes. The derived utility functions are presented subsequently, and the parameters used in the functions along with their descriptions and how their values for this study are generated are listed in Table 1. The alternative-specific parameters are generated based on the information provided by the service providers, or randomly (if the

Table 1. Parameters used in mode choice utility functions and their derivation in this study

Parameter	Description	Derivation
FLXP_TotalTT (min)	Flexpress total travel time (access time + in-vehicle time + egress time)	In-vehicle time is calculated based on the selected route. Access/egress time is calculated based on the selected mode and the distance between the boarding/alighting Flexpress terminal and the trip's origin/destination.
FLXP_TotalCost ($)	Flexpress total travel cost (access cost + fare + egress cost)	Fare is based on the selected route and will be changed in every iteration. Access/egress cost is based on the selected mode and the distance between the boarding/alighting Flexpress terminal and the trip's origin/destination.
Rail_TotalCost ($)	Rail total travel cost (access cost + fare + egress cost)	Fare is randomly generated between $20 and $50[a]. Access/egress cost is based on the selected mode and the distance between the boarding/alighting rail station and the trip's origin/destination.
GH_TotalCost ($)	Greyhound total travel cost (access cost + fare + egress cost)	Fare is randomly generated between $10 and $45[b]. Access/egress cost is based on the selected mode and the distance between the boarding/alighting Greyhound station and the trip's origin/destination.
Drive_TotalCost ($)	The fuel cost between trip origin and destination	Uses ADOT street network to find the shortest path between every trip's origin and destination, and considers a 1.2 travel time factor for peak periods. The cost is only the fuel cost, and an average cost of 17 cents/mi is considered.
FLXP_Hdwy (min)	Flexpress headway	Headway is based on the selected route and will be changed in every iteration.
Rail_Hdwy (min)	Rail headway	Randomly generated between 30 and 120 min[a]
GH_Hdwy (min)	Greyhound headway	180 min[c]
Gender	1 = female; 0 = male	Generated based on the distributions driven from the SP survey for each combination of OD and trip purpose.
Young	1:age < 35; 0:age ≥ 35	
Senior	1:age ≥ 60; 0:age < 60	
Occ_grp1	Occupation is accounting, administrative, insurance, education and teaching, or government	
Occ_grp2	Occupation is automotive, business, executive, manufacturing, sales, marketing, or real estate	
Occ_grp3	Occupation is engineering, design, IT, planning, media, or journalism	
Occ_grp4	Occupation is general labor, food services, or transportation	
Ride_to_St	Traveler drives to or is dropped off at the terminal/station	
PT_to_St	Traveler takes public transit to the terminal/station	
PT_from_St	Traveler takes public transit from the station/terminal to the final destination	
Places	Number of places visited during the trip	
Work	Trip purpose is work	Generated based on the distributions driven from the SP survey for each OD.
Event	Trip purpose is event	
Flight	Trip purpose is flight connection	
FFvisit	Trip purpose is visiting family and/or friends	
Commuter	Traveler travels between the two cities six times/year or more	
ICbusRides	Number of intercity bus rides the traveler has made in the last 10 years	
AvgDly (min)	Average delay the traveler previously experienced while driving between the two cities	
DlyPrcnt (%)	Occurrences of delays (as a percentage of total number of trips between the two cities) the traveler experienced	
Cars	Number of cars owned in the household	
FLXPsafetyRank	Traveler's impression of Flexpress safety: 1 = very low; 2 = low; 3 = moderate; 4 = high; 5 = very high	
AccessFactor	Accessibility of the service is one of the two most important factors to the traveler	
FreqFactor	Frequency of the service is one of the two most important factors to the traveler	

[a]Transit fares and service frequencies for passenger rail are not yet determined, so these values were generated randomly within a certain range.
[b]Due to the Greyhound fare policy ($1 for the first rider, increasing to as much as $45 for additional passengers).
[c]Based on information provided by Greyhound.

corresponding information is not available). For the personal- and trip-related parameters, the data distributions from the SP survey were used to generate the values. In the following equations, GH denotes Greyhound service and FLXP denotes Flexpress

$$U_{\text{FLXP}} = -0.0095 \times \text{FLXP\_TotalTT} - 0.0386 * \text{FLXP\_TotalCost} - 0.0106 \times \text{FLXP\_Hdwy} + 0.2923 \times \text{Gender} + 0.4907 \times \text{Occ\_grp2} + 0.3892 \times \text{Occ\_grp3} + 0.4927 \times \text{Work} + 0.7768 \times \text{Event} + 0.9476 \times \text{Flight} + 0.7053 \times \text{FFvisit} + 1.67162 \times \text{Ride\_to\_St} + 2.6952 * \text{PT\_to\_St} - 0.8257 \times \text{PT\_from\_St} - 0.6805 \times \text{Commuter} - 0.0020 \times \text{ICbusRides} + 0.0209 \times \text{DlyAvg} + 0.4103 \times \text{FLXPsafetyRank} + 1.5851 \times \text{AccessFactor}$$

$$U_{\text{Rail}} = 2.0408 - 0.0386 \times \text{Rail\_TotalCost} - 0.0106 \times \text{Rail\_Hdwy} + 1.5892 \times \text{AccessFactor}$$

$$U_{\text{GH}} = 4.5109 - 0.0386 \times \text{GH\_TotalCost} - 0.0106 \times \text{GH\_Hdwy} - 1.2109 \times \text{Gender} + 0.7637 \times \text{Occ\_grp4} - 1.2394 \times \text{Work} - 0.8989 \times \text{Places} + 2.0998 \times \text{PT\_to\_St} - 0.0360 \times \text{DlyPrcnt} - 3.5432 \times \text{FreqFactor}$$

$$U_{\text{Drive}} = 2.1117 - 0.0386 \times \text{Drive\_TotalCost} + 0.4748 \times \text{Young} - 1.4178 \times \text{Senior} - 0.7495 \times \text{Occ\_grp1} + 0.3544 \times \text{Cars} + 0.0567 \times \text{Places} - 0.5187 \times \text{FLXPsafetyRank} + 1.7057 \times \text{AccessFactor}$$

Fast-Trips: Dynamic Transit Passenger Assignment Model

What is Fast-Trips?

Fast-Trips, standing for **F**lexible **A**ssignment and **S**imulation **T**ool for **Tr**ansit and **I**ntermodal **P**assenger**s**, is an open-source modeling tool for dynamic transit passenger assignment, originally developed at the University of Arizona and the University of Texas at Austin (Khani et al. 2015; Khani 2013). It uses a trip-based hyperpath (TBHP) model to generate a set of paths with low generalized cost. TBHP is a stochastic path set generation algorithm, because each hyperlink represents a number of actual links which are chosen probabilistically when paths are enumerated. TBHP can be formulated as a frequency-based or schedule-based model, but the current version of Fast-Trips applies the TBHP only to a schedule-based network.

In 2014, three agencies, the Metropolitan Transportation Commission (MTC), the San Francisco County Transportation Authority (SFCTA), and the Puget Sound Regional Council (PSRC), received a grant to extend this research to develop and implement Fast-Trips for travel demand forecasting and analyzing transportation investments (MTC, SFCTA, and PSRC 2018). This enhanced version of Fast-Trips incorporates fares; considers heterogeneity of passenger demographics; captures the effect of boardings, alightings, and crowding on transit vehicle dwell times; measures the effect of transit service on the passenger experience (e.g., waiting longer to get a seat, or riding a few stops in the wrong direction to get a seat on a crowded line); and considers the effect of missed transfers and travel time reliability on people's perceptions of the quality of transit (Zorn and Sall 2017). This project implemented a production-ready and calibrated person-based dynamic transit assignment component in a regional transportation planning model, which will help to improve many regional transit-related projects.

The input to Fast-Trips consists of a transit network directory, a transit demand directory, and Fast-Trips configuration. The files in the transit network directory are specified by a transit network data standard that is suitable for dynamic transit modeling and is based on Google's General Transit Feed Specification (GTFS), called GTFS-Plus (Coe et al. 2018). GTFS is a standard text-based format for transit network data provided by transit agencies and shared publicly by Google in many metropolitan areas (GTFS 2018). It contains very detailed information about the network, in which each route has a set of vehicle trips, and each vehicle trip includes a list of the stops and the scheduled arrival and departure times for each stop. For each stop, there is information about the location and the type of stop, and the calendar indicates the service provided on each day of the week. The files in the transit demand directory are specified by the Dyno-Demand data standard, a travel demand data standard that is suitable for dynamic transit modeling and that contains information about passenger trips to be assigned, as well as person-level and household-level information about passengers (Sana et al. 2018). The required and optional files for the two aforementioned data standards as well as the configuration files are listed in Table 2.

Preparing GTFS-Plus and Dyno-Demand Files for Flexpress

The agency.txt, calendar.txt, routes.txt, routes_ft.txt, stops.txt, and vehicles_ft.txt files are created using Flexpress service characteristics and the outputs of the TNDFSP model are explained in section "Case Study." The trips.txt, trips_ft.txt, and stop_times.txt files are created using the route frequencies and travel time between each two terminals along a route, which also are outputs of the TNDFSP model. Fare-related files (fare_periods.txt, fare_attributes_ft.txt, and fare_rules.txt) are created based on the fare policy and price considered for Flexpress, but as fares change in every iteration, these files are updated.

The access/egress modes considered in this study were walking, driving [Park-and-Ride (PNR)] and pick-up/drop-off [Kiss-and-Ride (KNR)], and the corresponding access files (walk_access_ft.txt, drive_access_ft.txt, and drive_access_points_ft.txt) were created using the distance between traffic analysis zones (TAZs) and Flexpress terminals. It was assumed that all the Flexpress terminals have a park-and-ride facility and that travelers can drive to any terminal in the city. However, a terminal was considered to be accessible from/to a TAZ on foot only if it takes 15 min or less to walk from a TAZ to a terminal (or vice versa). Assuming an average walking speed of 1.4 m/s (3.1 mph), the corresponding distance between the terminal and the TAZ was 1.3 km (0.8 mi) or less. Since Flexpress is an intercity service with no transfer between routes, the required transfers.txt is an empty file, but transfer links between PNR lots and Flexpress terminals are accounted for in transfers_ft.txt.

Because the Flexpress network used for this study was designed using the demand for the morning peak period, the demand used in this study also was for that period. For each of the 124,571 morning person-trips between Tucson and Phoenix, a record was generated that included all the required parameters for mode choice utility functions (Table 1), as well as those required for the Fast-Trips trip_list.txt file. These 124,571 records were stored in a demand file named demand_list.txt.

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Table 2. Inputs to Fast-Trips

File name	Description	Required/optional	Considered in this study?
	Network files (GTFS-Plus data standards)		
agency.txt	Contains a record for each transit agency, including name, URL, and time zone	Required	Yes
calendar.txt	Contains a record for each service category used in trips.txt, including start/end date and days of the week	Required	Yes
walk_access_ft.txt	Contains a record for each TAZ to accessible transit stops, as well as a record for each stop to all accessible TAZs, including TAZ, stop ID, direction (access/egress), and distance	Required	Yes
bike_access_ft.txt	Contains a record for each transit stop that can be biked to from each TAZ, including TAZ, stop ID, and distance	Optional	No
drive_access_ft.txt	Contains a record for each PNR/KNR that can be driven to from each TAZ, including TAZ, lot ID, direction (access/egress), distance, cost, and travel time	Optional	Yes
drive_access_points_ft.txt	Contains a record for each drive access point (e.g., PNR lots and KNR drop-off areas), including lot ID and lot latitude and longitude	Optional[a]	Yes
transfers.txt	Contains a record for each pair of transit stops that can be transferred between on foot, including from/to stop ID and transfer type	Required	Yes
transfers_ft.txt	Contains a record for each pair of transit stops, or PNR/KNR and transit stop that can be transferred between on foot, including from/to stop ID and distance	Optional[a]	Yes
vehicles_ft.txt	Contains a record for each vehicle type, including vehicle name, seated and standing capacity, and maximum speed	Required	Yes
routes.txt	Contains a record for each transit route, including route ID and service type	Required	Yes
routes_ft.txt	Contains a record for each transit route, including route ID and mode	Required	Yes
trips.txt	Contains a record for each transit vehicle trip, including trip ID, route ID, and service ID	Required	Yes
trips_ft.txt	Contains a record for each transit vehicle trip, including trip ID and vehicle name	Required	Yes
stops.txt	Contains a record for each transit stop, including stop ID, stop name, and stop latitude and longitude	Required	Yes
stop_times.txt	Contains a record for every scheduled stop within a trip, including trip ID, arrival/departure time, stop ID, and stop sequence	Required	Yes
fare_attributes_ft.txt	Contains a record for each fare type, including fare period, price, currency type, payment method, and transfer information	Optional	Yes
fare_rules.txt	Specifies how fares in the fare_attributes_ft.txt apply to an itinerary by origin/destination stop, zones, or route, including fare ID and route ID	Optional	Yes
fare_periods_ft.txt	Adds start and end times to fare rules, including fare ID, fare period, and start/end time	Optional	Yes
	Demand files (Dyno-Demand data standards)		
trip_list.txt	Contains a record for each passenger trip to be assigned, including origin/destination TAZs, mode, purpose, departure/arrival time, time target, and value of time	Required	Yes
person.txt	Contains a record for persons taking a trip, including person-level variables such as age, gender, worker status, and transit pass	Optional	No
household.txt	Contains a record for households with a person taking a trip, including household-level variables such as income, vehicles, number of people, and number of workers	Optional	No
	FT configuration		
pathweight_ft.txt	Tells FT how much to value each attribute of a path, including user class, purpose, demand mode, supply mode, and weight value	Required	Yes
config_ft.txt	Determines configuration settings for FT run and pathfinding	Required	Yes
config_ft.py	Contains functions that are evaluated to ascertain items such as user classes, and can be used to define user classes based on person, household, and/or trip attributes	Optional	No

Note: FT = Fast-Trips.
[a]Required if drive_access_ft.txt is present.

The required parameters for trip_list.txt include origin and destination TAZs, travel mode, purpose, desired departure and arrival times, time target (indicating which of arrival or departure time is more important to the trip maker), and value of time. The purpose and travel mode are mutual between trip_list.txt and mode choice utility functions, and are derived as described in Table 1; however, travel mode in trip_list.txt is a combination of access mode, transit mode, and egress mode (e.g., KNR-Rapid_Bus-Walk). For the sake of simplicity, the departure and arrival times were randomly generated within the specified period (by drawing from a uniform random distribution), and for all the trips, time target was set to departure. Value of time was obtained from Table 4 of the USDOT report on the value of travel time savings (US Department of Transportation 2015), which presents average values of time for different trip purposes for intercity surface travel modes.

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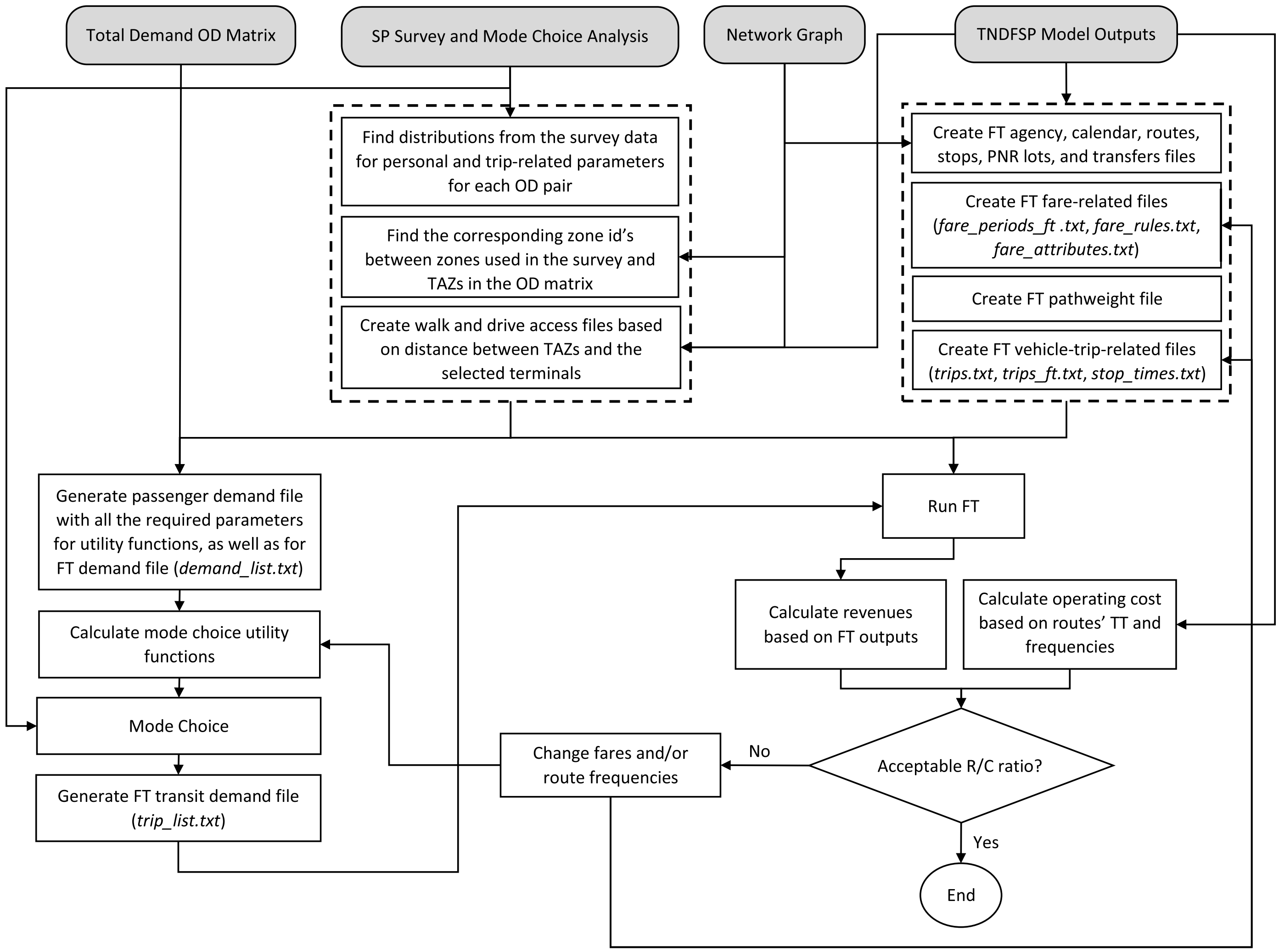

Fig. 2. Detailed solution framework implemented in this study. FT = Fast-Trips.

Once mode choice is conducted, trip_list.txt is created by copying the records from demand_list.txt that are assigned to Flexpress to a new file, and by excluding all the parameters related to mode choice utility functions. The other two Dyno-Demand files (person.txt and household.txt) are optional and were not considered in this study.

Calculating Revenue and Operating Cost

The cost considered in this study was only the operating costs, and capital costs such as building terminals and purchasing transit vehicles were not considered explicitly. The operating cost for Flexpress was the sum of energy, maintenance, and labor costs. Considering $1.12/km ($0.7/mi) for energy and maintenance cost for battery electric buses (National Renewable Energy Laboratory 2017), an average speed of 126 km/h (78.5 mph) for Flexpress [250 km/h (150 mph) on the dedicated freeway section and 32 km/h (20 mph) in the urban areas], and $30/h for labor cost (including wages and fringe benefits) (Kay et al. 2011), the operating cost for Flexpress was $85/h: $0.7 \times 78.5 + 30 = 85$. Revenue was calculated based on the Fast-Trips results, which show the route(s) that each passenger has taken and the fare that each passenger has paid. The R/C ratio was calculated by dividing revenue by operating cost.

Solution Framework

The solution framework implemented in this study is composed of the following steps (Fig. 2):

1. Create demand_list.txt as explained in subsection "Preparing GTFS-Plus and Dyno-Demand Files for Flexpress."
2. Convert the transit network and service characteristics to GTFS-Plus files (agency.txt, calendar.txt, transfers.txt, vehicles_ft.txt, routes.txt, routes_ft.txt, trips.txt, trips_ft.txt, stops.txt, stop_times.txt, fare_periods.txt, fare_attributes_ft.txt, and fare_rules.txt) as explained in subsection "Preparing GTFS-Plus and Dyno-Demand Files for Flexpress."
3. Set weights for every component of a transit trip, considering different trip purposes (pathweight_ft.txt).
4. Generate access files (walk_access_ft.txt, drive_access_ft.txt, and drive_access_points_ft.txt) as explained in subsection "Preparing GTFS-Plus and Dyno-Demand Files for Flexpress."
5. Calculate operating costs based on the total vehicle hours traveled in the system and the per-hour operating cost, as explained in section "Calculating Revenue and Operating Cost."
6. Assign the available OD travel demands to modes using the mode choice model and the utility functions presented in section "Mode Choice Model." It is assumed that the total travel demand and the attributes of competing modes (passenger rail, Greyhound, and drive) remain the same in every iteration.

7. Create trip_list.txt from demand_list.txt, as explained in subsection "Preparing GTFS-Plus and Dyno-Demand Files for Flexpres."
8. Run Fast-Trips to assign Flexpress trips to the different routes in the Flexpress network.
9. Calculate revenue based on the Fast-Trips results [the route(s) that each passenger has taken and the fare that each passenger has paid] for passengers who successfully boarded the service.
10. If the ratio of revenue to operating cost (R/C ratio) reaches the desired value, go to Step 13.
11. Change the service frequencies and/or fares based on the route usage in the Fast-Trips results (subsection "Scenarios and Model Results").
12. Update fare-related files (fare_periods.txt, fare_attributes_ft.txt, and fare_rules.txt) based on the new fares, and update vehicle-trip-related files (trips.txt, trips_ft.txt, and stop_times.txt) based on the new frequencies and the resulted service schedules. Then go back to Step 5.
13. Stop.

Results and Discussion

Scenarios and Model Results

To study the effects of different fares and frequencies on the system performance, four frequency designs and two fare policies were considered (Table 3). First, the frequencies obtained from the TNDFSP model, referred to as design-oriented frequencies, were used. Then, one scenario in which the minimum frequency (frequency = 4) was considered for all routes and two scenarios in which routes with high or moderate demand are assigned higher frequencies were tested. High-demand routes in this context were routes for which many passengers could not board the bus in the previous iteration, and moderate-demand routes were those for which transit vehicles reached their capacity. The frequencies in the latter two scenarios, which were determined based on the system usage and ridership, are referred to as service-oriented frequencies. Additionally, two fare policies were considered: one with a flat fare for all routes, and one in which routes with high frequencies (frequency ≥ 10) had a higher fare.

The solution method described in the previous section was run for various combinations of fares and frequencies for the Flexpress service, and the results are presented in Table 4. Flexpress market share and demand decreased with an increase in fares and a decrease in frequencies, which is intuitive. The total Flexpress demand was very sensitive to fares, whereas the number of boarded passengers on Flexpress routes had a low sensitivity to fares. This occurred because there is no competing mode in the assignment model (Fast-Trips), and the model assigns passengers to the path with the lowest generalized cost. Conversely, in the mode choice model, there is a competition between modes to obtain a higher portion of demand, and travel cost is one of the highly influential factors.

The sensitivity of the model in terms of various service performance measures is further discussed in the next subsection, but a general observation from the results is that the scenarios with design-oriented frequencies (D1) had lower R/C ratios than those in which frequencies were set based on route usage (D3 and D4). The assignment results showed that the TNDFSP model overestimated demand for some routes, and that those high frequencies were not required. In contrast, demand was underestimated for some other routes, and low frequencies and the consequent limited capacity resulted in many passengers being unable to board their chosen routes. These differences in the route assignment occurred because the Flexpress network was designed based on a transit demand matrix that was formed from 5% of total travel demand for each OD, but in the current framework, the transit demand is derived from the mode choice model, which has no constraint for area or OD coverage. As a result, some people traveling between certain ODs (e.g., ODs with shorter distances or those in which either the origin or the destination is not close enough to a Flexpress terminal) may choose another mode, and so the usage of routes will be different than that predicted in the network design model.

In all scenarios, a portion of transit demand was not served (Table 4). As explained previously, this partly was because of the limited capacity that resulted from insufficient frequencies. Scenarios with service-oriented frequencies (frequency = $\{4, 10\}$ and frequency = $\{4, 6, 12\}$) had higher numbers of boarded passengers and higher satisfied transit demand ratios than did scenarios with design-oriented frequencies. The second reason for unsatisfied demand was the limited period (the 3 h of the morning peak) considered for the assignment, which resulted in passengers at the end of the period being unserved rather than assigned to a later service. For example, if the last bus for the route that a passenger has chosen leaves at 8:50, and the passenger's departure time is 8:45 but it takes them 7 min to get to the terminal, they will miss the bus and will not be served. The third reason is related to access/egress modes. People know how they would get from home to the boarding terminal (e.g., walk, drive, taxi, and so forth), and from the alighting terminal to their ultimate destination; therefore, passengers' access/egress modes in Fast-Trips are predetermined for each trip. However, not all terminals are accessible on foot from a TAZ. Therefore, if passengers are limited to walking for the access or egress mode, and there is no accessible terminal at one or both of their trips ends, they will not be served. This limitation goes

Table 3. Scenarios considered for different frequency designs and fare policies

Category	Scenario	Description	Notion
Frequency design	D1	Frequencies obtained from the network design model are used, which are the set of $\{4, 5, 6, 8, 10, 21, 26\}$.	Frequency = design-oriented
	D2	Minimum frequency is considered for all routes.	Frequency = 4
	D3	Frequency = 10 for high-demand routes; Frequency = 4 for all other routes	Frequency = $\{4, 10\}$
	D4	Frequency = 12 for high-demand routes; Frequency = 6 for moderate-demand routes; Frequency = 4 for all other routes	Frequency = $\{4, 6, 12\}$
Fare policy	P1	Flat fare is considered for all routes.	Fare = 10, 15, 20, 25, 30
	P2	Regular fare ($10 or $20) for routes with frequency < 10; Increased fare ($20 or $30) for routes with frequency ≥ 10	Fare = $\{10, 20\}$; Fare = $\{20, 30\}$

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Table 4. Flexpress system performance for various combinations of frequency designs and fare policies

Frequency	Fare	FLXP share	FLXP demand	Boarded passengers	Satisfied FLXP demand ratio	Revenue ($)	Operating cost ($)	R/C ratio
Design-oriented	10	7.73	9,630	5,737	0.60	57,370	60,527	0.948
	15	7.69	9,580	5,738	0.60	86,070		1.422
	20	7.59	9,454	5,732	0.61	114,640		1.894
	$\{10,20\}$	7.73	9,630	5,737	0.60	57,370		0.948
	25	7.43	9,249	5,722	0.62	143,050		2.363
	30	7.12	8,868	5,687	0.64	170,610		2.819
	$\{20,30\}$	7.59	9,454	5,732	0.61	114,640		1.894
4	10	7.73	9,630	5,667	0.59	56,670	40,876	1.386
	15	7.69	9,576	5,668	0.59	85,020		2.080
	20	7.58	9,445	5,667	0.60	113,340		2.773
	25	7.41	9,229	5,657	0.61	141,425		3.460
	30	7.09	8,828	5,620	0.64	168,600		4.125
$\{4,10\}$	10	7.73	9,630	6,423	0.67	64,230	46,551	1.380
	15	7.7	9,593	6,434	0.67	96,510		2.073
	20	7.6	9,460	6,425	0.68	128,500		2.760
	$\{10,20\}$	7.69	9,576	6,421	0.67	64,210		1.379
	25	7.43	9,252	6,423	0.69	160,575		3.449
	30	7.1	8,848	6,371	0.72	191,130		4.106
	$\{20,30\}$	7.52	9,370	6,423	0.69	128,460		2.760
$\{4,6,12\}$	10	7.73	9,633	7,417	0.77	74,210	53,737	1.381
	15	7.71	9,598	7,445	0.78	111,675		2.078
	20	7.61	9,473	7,435	0.78	148,700		2.767
	$\{10,20\}$	7.73	9,633	7,417	0.77	74,210		1.381
	25	7.45	9,283	7,378	0.79	184,450		3.432
	30	7.18	8,943	7,334	0.82	221,550		4.123
	$\{20,30\}$	7.61	9,473	7,435	0.78	148,700		2.767

back to the mode choice model, which does not consider access to transit in the utility functions. In other words, ignoring accessibility in the mode choice model results in passengers being assigned to Flexpress even though due to long access distances they cannot actually be served by Flexpress.

The R/C ratio increases with an increase in fares and a decrease in frequencies. The sensitivity of revenue and R/C ratio to fares and frequencies are further discussed in the next subsection, but a general observation is that if frequencies are designed efficiently (service-oriented frequencies), the system will be profitable, and with an increase in fares, operators can achieve even higher R/C ratios. The capital costs were not considered explicitly in this study; however, there are cases with R/C ratios of 3 or 4, which implies that the service might be able to account for the capital costs as well.

Sensitivity Analysis

Fig. 3 shows the sensitivity of model to various combinations of fare and frequencies, in terms of demand, boarded passengers, revenue, and the R/C ratio.

Flexpress demand decreased with an increase in fares and a decrease in frequencies; however, although the demand was very sensitive to fares, its sensitivity to frequencies was insignificant [Fig. 3(a)]. This is because the Flexpress utility function in the mode choice model is about four times more sensitive to cost than to headways.

The number of boarded passengers on Flexpress routes, however, was exactly the opposite, with low sensitivity to fares and high sensitivity to frequencies [Fig. 3(b)]. As mentioned in the previous subsection, the insensitivity to fares is due to the absence of competing modes in the assignment model. Regarding frequencies, it was shown that systems with service-oriented frequencies better satisfy the demand than do those with design-oriented frequencies. This indicates that the system does not always perform as planned, and it is important to modify the frequencies (and generally the design) based on the system usage and ridership. Furthermore, among systems with service-oriented frequencies, those with the frequency set $\{4,6,12\}$ served higher demand than those with the frequency set $\{4,10\}$, which indicates that more careful frequency determination in response to system ridership will result in a better performance [Fig. 3(b)].

Revenue increased with an increase in frequencies, which was due to the direct influence of service frequency on ridership [Fig. 3(c)]. Revenue also increased with an increase in fares, which is intuitive.

Fig. 3(d) exhibits the results of R/C ratio. Revenue increased with an increase in frequency, but so did the operating cost, and as a result, the R/C ratio did not change much for different frequencies. This implies that there are multiple ways to achieve a certain R/C ratio, and therefore it is the other objectives that define the final design and service characteristics. For example, the minimum-frequency scenario (D2) and the two scenarios with service-oriented frequencies (D3 and D4) resulted in about the same R/C ratios across different fares. If an operator wishes to serve more passengers, the D4 scenario (frequency $= \{4,6,12\}$) is the best one, whereas if the objective is to minimize the operating costs, the D2 scenario (frequency $= 4$) should be selected. Another interesting result in Fig. 3(d) is that, in terms of R/C ratio, the scenarios with service-oriented frequencies significantly outperformed the one with design-oriented frequencies. This emphasizes the importance of considering elastic demand in a transit network design, and indicates that a less efficient system may result if the elasticity of transit demand to service characteristics is not considered.

The effect of the two fare policies (P1 and P2) on the system performance is shown in Fig. 4. Because the number of high-frequency routes (for which a higher fare was applied in

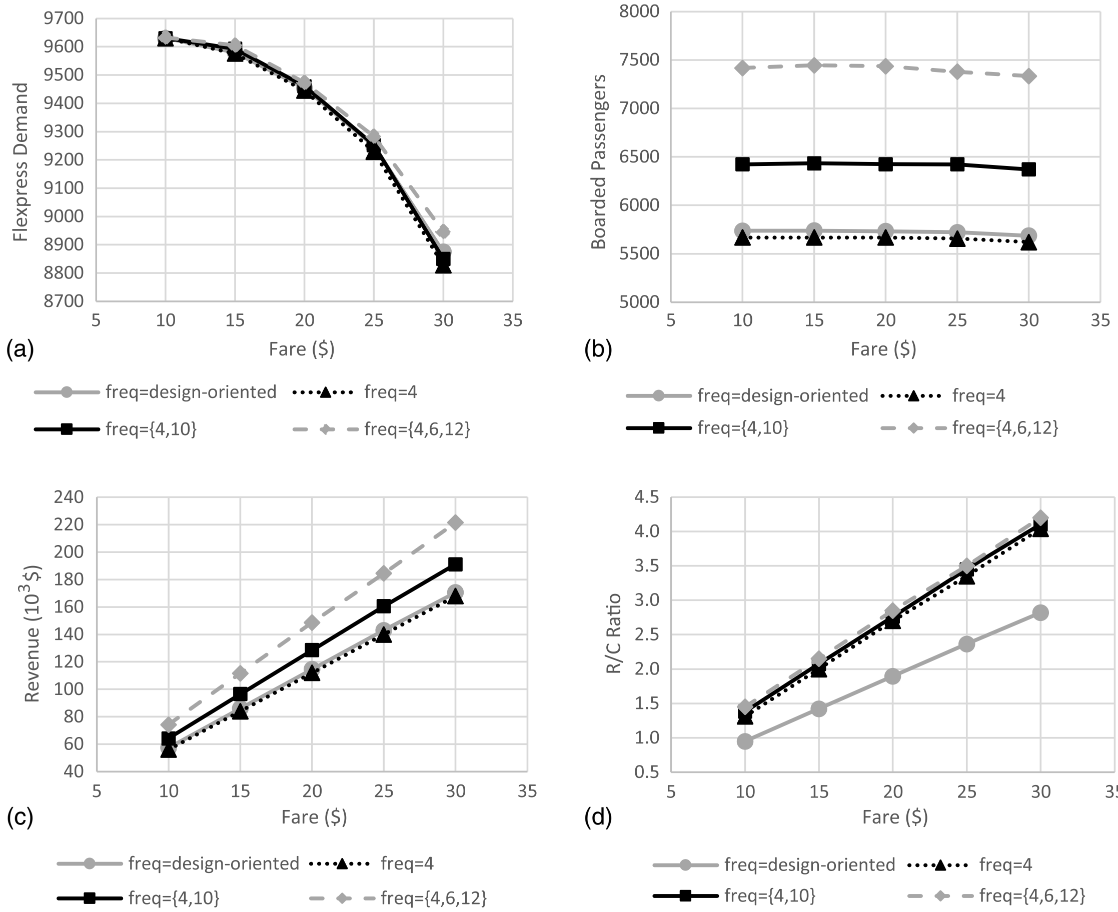

Fig. 3. Results of various combinations of frequency designs and the flat-fare policy on Flexpress service performance in terms of (a) Flexpress demand; (b) boarded passengers; (c) revenue; and (d) R/C ratio.

the P2 scenario) was limited (1–5 of 30 routes), and the assignment was not very sensitive to fares, not much change occurred between the two fare policies across different frequency designs. In most cases, the results of the P2 scenario were similar to those of the P1 scenario, in which the P1 flat fare was equal to the P2 regular fare (e.g., P1, with fare $= 10$, and P2, with fare $= \{10, 20\}$).

Due to the limited number of high-frequency routes, even demand (which is the performance measure most sensitive to fares) remained almost the same between the two fare policies in most cases. The only case in which an obvious change occurred in demand between the P1 and P2 scenarios was the case with frequency $= \{4, 10\}$ (Scenario D3), wherein the number of high-frequency routes (frequency ≥ 10), and consequently the number of routes with an increased fare, were higher than in the other cases [Fig. 4(a)]. As a result, demand in the D3 + P2 cases was lower than that in the corresponding D3 + P1 cases, in which the P1 flat fare was equal to the P2 regular fare. However, due to the average increased fare, even in these cases, the revenue and consequently the R/C ratio were the same between the corresponding P1 and P2 cases.

Conclusion

This study integrated a dynamic transit passenger assignment tool with a mode choice model and a transit service design module in an iterative approach, and presented a solution framework for re-optimizing the service design (fares and frequencies) considering elastic transit demand. The proposed solution framework has high resolution to passenger behaviors, provides a powerful tool for operational and financial assessment of transit services, and can be applied to any transit service, whether urban or intercity.

As a case study, the proposed solution framework was applied to the network of Flexpress, a newly conceived intercity transit service between the metropolitan areas of Tucson and Phoenix, Arizona. The ultimate service design depends on the objective set by the system planners, but it was shown that in any case if the elastic transit demand is not considered in the network design, a less efficient system results. For example, if the desired R/C ratio is set to 3 or more and the operators wish to serve as many passengers as possible, an efficient service design for the predefined route network entails the frequency set $\{4, 6, 12\}$ and a flat fare of \$25 for all routes. In this design, from the 30 Flexpress routes, two would operate at a frequency of 12 bus/h, 15 would have a frequency of 6 bus/h, and the other routes would operate at the minimum frequency of 4 bus/h, resulting in 7,378 boarded passengers, \$53,737 operating cost, and \$184,450 revenue ($R/C = 3.43$). When the elasticity of transit demand to service characteristics was not considered, the optimal design found for the Flexpress network included two routes with a frequency of 20 or more bus/h, seven with a frequency of 5–10 bus/h, and the rest operating at a frequency of 4–9 bus/h. Applying the same fare (\$25) and about the same transit demand to the latter network resulted in a less efficient system with ~1,700 fewer passengers served, ~\$6,800 more operating cost, and ~\$41,000 less revenue.

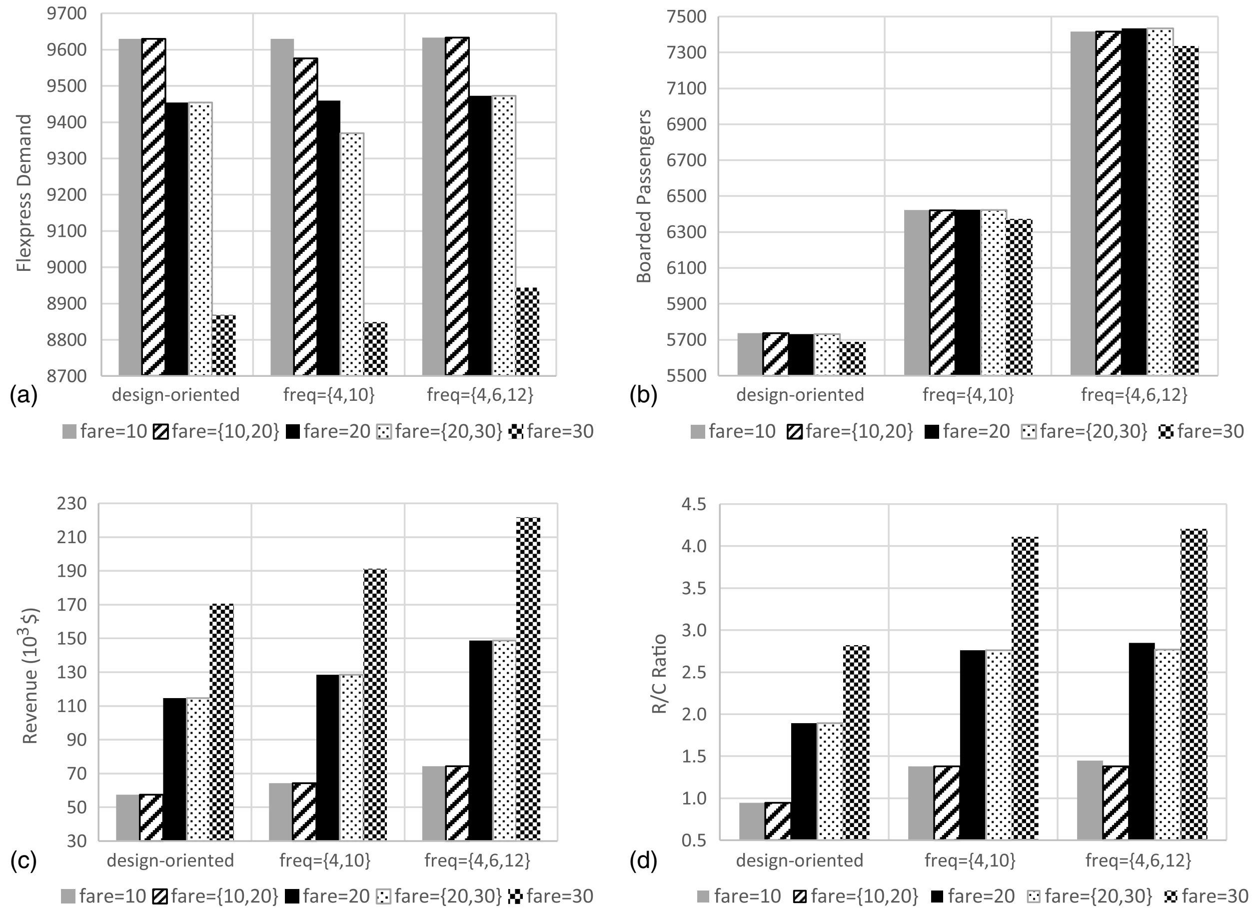

Fig. 4. Results of various combinations of frequency designs and fare policies on Flexpress service performance in terms of (a) Flexpress demand; (b) boarded passengers; (c) revenue; and (d) R/C ratio.

The solution framework can be employed to study the effect of different fare structures (such as time-based, distance-based, or zone-based fares) or different fare policies (such as offering certain discounts for transit-pass holders, or applying an integrated fare system in which the method of payment is the same for all modes and services and passengers can easily switch between services). In another application, the passenger assignment model (Fast-Trips) can be linked with a traffic simulation tool or an operation algorithm that models the operation of autonomous vehicles (AVs) and/or shared-mobility services [transportation network companies (TNCs)], such as Lyft and Uber, to integrate them as access/egress modes for public transit and to assess the operational performance and financial viability of the integrated AV + PT and/or TNC + PT systems, which are believed to dominate the future of transportation.

There also are certain ways to extend the current study. This research focused on the ratio of revenue to operating cost (R/C ratio) as the design objective, but this criterion does not consider equity and area coverage, which are very important for public transportation services. As an alternate study, the proposed solution framework can be applied to other design objectives, such as providing enough area coverage or a desirable equity, achieving a certain level of ridership, or even a weighted sum of these and the R/C ratio. This study also can be extended by expanding the service period to a full day, which provides a basis for interesting future research. For example, one can consider various service plans and time-based fares for the peak and off-peak periods to determine the likely ridership and revenue of the system. Another possible extension would be to model the first/last mile more accurately. This study considered only walking and drive-based modes as access and egress modes for intercity transit, but they can be expanded by incorporating the local transit [bus, streetcar, and light rail transit (LRT)] as well. This would be easy to model because Fast-Trips reads transit network specifications in a GTFS-based format, and the GTFS files are available for most metropolitan areas. Once that integration is in place, in addition to the characteristics of the intercity service, one can also study the effect of local services' fares and frequencies as well as the intermodal fare policies and operation management (e.g., transfer coordination) on the system ridership and revenue.

Moreover, the results of this study showed that if the transit network is designed separately, the design-oriented frequencies may not work well in the actual operation. This emphasizes the importance of considering elastic transit demand, and indicates that to achieve a more efficient system, it is better to modify service characteristics (fares, frequencies, and so forth) based on the passenger path choice behaviors and route usage. The same situation may hold for other aspects of network design, such as the number and location of terminals and the number and configuration of routes. Therefore, an interesting future research direction will be to incorporate the network design model in the iterative process as well, such that, under elastic demand considerations, the network (routes, terminals, and frequencies) will be designed based on the R/C ratio or any other objective.

Data Availability Statement

Some data, models, and code used during the study were provided by a third party (total demand OD matrix, statewide network graph, and Fast-Trips model). Direct requests for these materials may be made to the provider as indicated in the Acknowledgements. Some data, models, and code generated or used during the study are available from the corresponding author by request (mode choice model, Flexpress GTFS-Plus and Dyno-Demand files, and conversion scripts for Fast-Trips input/output files).

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